Section Q

Mathematical Modeling of Biological Neurons

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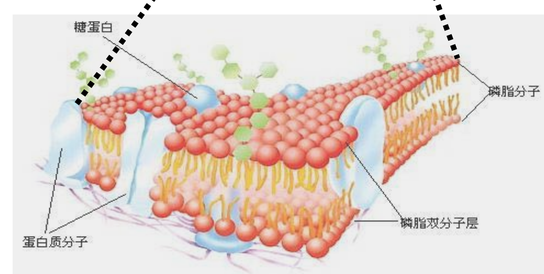
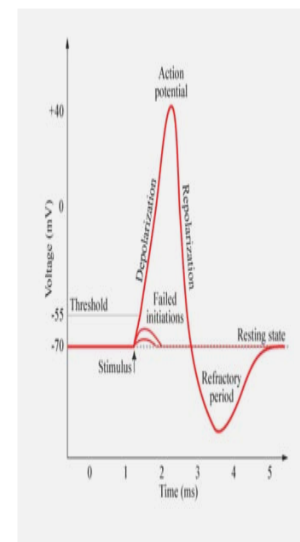
Cerebral Cortex and Neurons

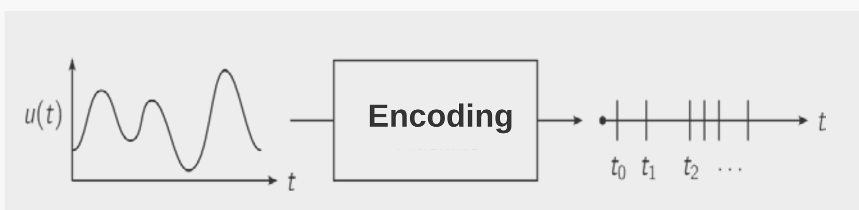
- 10^{11} neurons and 10^{15} connections, ~10^{4} neurons per mm^{2}, shape and functions

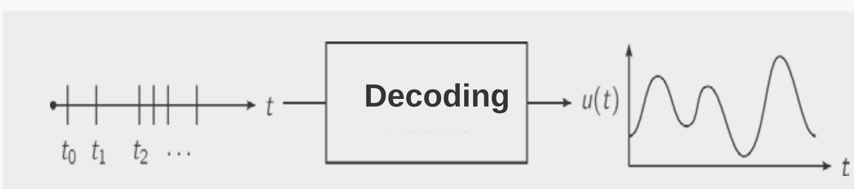




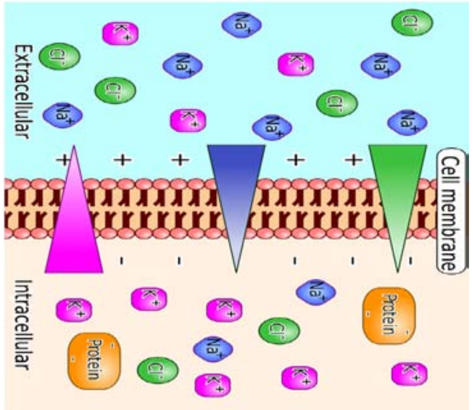
$$V_m(t) = V_{in}(t) - V_{out}(t)$$

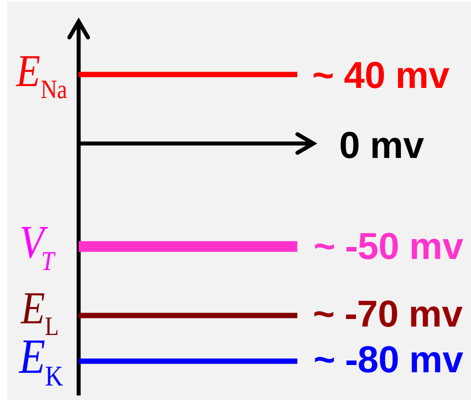
Information encoding and decoding



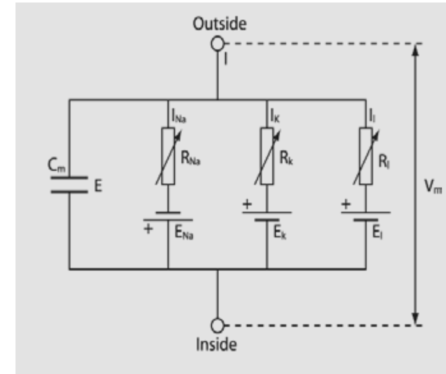


Hodgkin-Huxley (HH) model



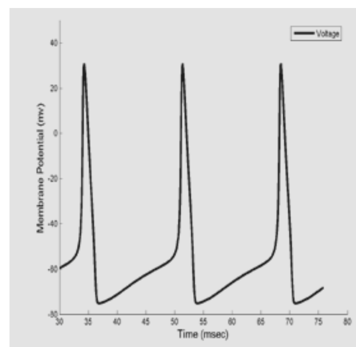


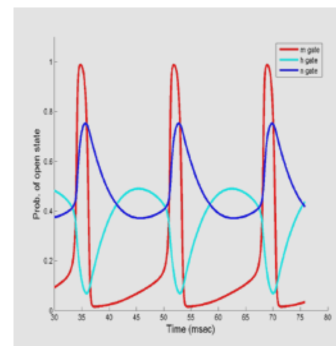
modeling

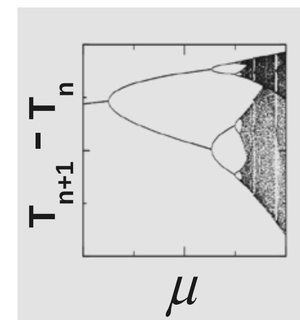


$$\begin{cases} C\dfrac{dV}{dt} = -G_L\left(V - E_L\right) - G_{\mathrm{Na}} m^3 h\left(V - E_{\mathrm{Na}}\right) - G_K n^4\left(V - E_K\right) + I^{\mathrm{external}}(t) \\ \dfrac{dm}{dt} = \alpha_m(1-m) - \beta_m m = \dfrac{1}{\tau_m(V)}\left(m_\infty(V) - m\right) \\ \dfrac{dh}{dt} = \alpha_h(1-h) - \beta_h h = \dfrac{1}{\tau_h(V)}\left(h_\infty(V) - h\right) \\ \dfrac{dn}{dt} = \alpha_n(1-n) - \beta_n n = \dfrac{1}{\tau_n(V)}\left(n_\infty(V) - n\right) \end{cases}$$

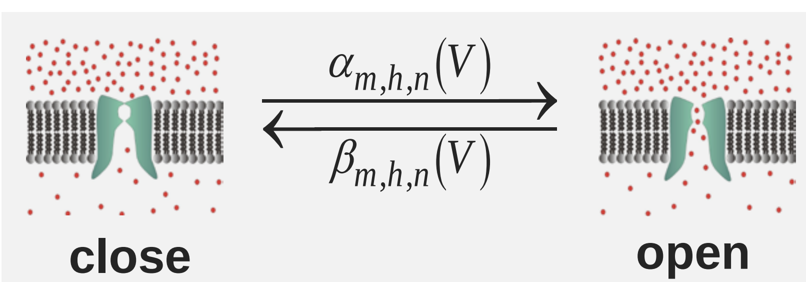
e.g. $I^{\mathrm{external}}(t) = I_0 + I_1 \sin(2\pi\mu t)$

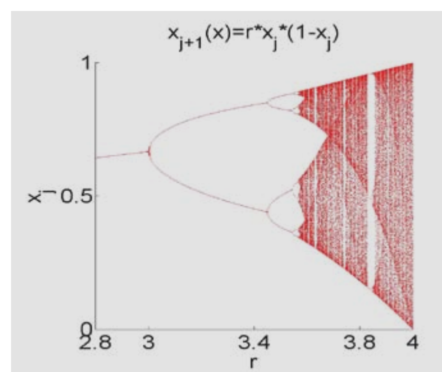






gating variables: $m,\ h,\ n$





Logistic map

$$x_{j+1} = r x_j\left(1 - x_j\right)$$