Dynamics and Fluctuations in Collective Motion

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SJTU: X. Chen, X. Yang, H. Li, C. Tai, X. Dong, D. Cai
Univ. of Texas: A. Be’er, E. L. Florin and Harry L. Swinney
Collective Motion in a Fish School
Collective Motion in a Bird Flock

Flocks of European Starlings near Oxford, Britain.
Quantify Bird Motion in Flocks

Stereo Photography:
match images from different view angles
positions of thousands of birds in a flock

Cavagna, Giardina, et al.  

Statistical physics is for the birds  
*Physics Today* (2007)
From Disorder to Order in Marching Locusts

6 minutes (accelerated 6 times) of video recording of an experiment with 120 locusts

Buhl, et al.
*Science* 2006
Collective Motion in Epithelial Cell

Collective cell migration is important for wound healing and cancer cell spreading.


Collective Motion of Vibrated Polar Disks

Physical interactions:
Inelastic collision leads to alignment

Collective Motion in Driven Filaments


Go With Neighbors: a “Ferromagnetic” Flock

Numerical models
Controlling parameters:
→ Strength of alignment;
→ Noise;
→ Density.

Viscek et al. PRL (1995);
Ballerini, et al. PNAS (2008);
Ginelli and Chate, PRL (2010).

Continuum theories:

\[
\rho(\vec{r},t) = \left\langle \sum_{\alpha} \delta(\vec{r} - \vec{r}_{\alpha C}(t)) \right\rangle \quad \text{concentration}
\]

\[
\tilde{P}(\vec{r},t) = \frac{1}{\rho(\vec{r},t)} \left\langle \sum_{\alpha} \hat{v}_{\alpha} \delta(\vec{r} - \vec{r}_{\alpha C}(t)) \right\rangle \quad \text{polar order}
\]

\[
Q_{ij}(\vec{r},t) = \frac{1}{\rho(\vec{r},t)} \left\langle \sum_{\alpha} \left( \hat{v}_{\alpha i} \hat{v}_{\alpha j} - \frac{1}{3} \delta_{ij} \right) \delta(\vec{r} - \vec{r}_{\alpha C}(t)) \right\rangle \quad \text{nematic order}
\]

“Ferromagnetic” Flock:
→ Off-lattice
→ Topology depends on velocities

Liquid crystal physics with active driving
Ramaswamy, Toner, Joanny, Prost, Marchetti ……
Common Features of Collective Motion

Biology:
→ Widely existing biological phenomenon;
→ Important biological functions.

Physics:
→ Large number of self-propelled objects;
→ Convert external energy into motion;
→ Local interactions;
→ Intrinsic noises cause fluctuations;
● Non-equilibrium systems;
● Statistically steady-state;

For more experiments and theories:
Hydrodynamics of a Swimming Bacterium

Reynolds number:

\[ R_e = \frac{V L}{\mu} \ll 1 \]

Taylor, Lighthill, Purcell, Avron, Lauga, Pedley, Powers, Golestanian, Yeomans ........
A Macro-Scale Bacterium Model

Motor & gear box

Rigid helix

viscous fluid: $\mu = 10^5 \mu_{water}$
$D = 2.5 \text{ cm}; f = 0.1 \text{ Hz}$
$Re = \frac{VL}{\mu} \sim 10^{-3}$

Mechanical Wave

Fluid motion

“Bacterium” Motion
Bacteria Colonies on Agar Gel Substrates

Agar Gel Substrate

Extra-cellular matrix + bacteria

Paenibacillus dendritiformis

Agar matrix, Water, and Nutrients
Paenibacillus dendritiformis Colony Growth in 6 Days
Collectively Moving *B. subtilis* at Low Density

Track more than 95% of all bacteria in the field of view:

Vector color $\rightarrow$ dynamic cluster

Clusters containing more than two bacteria are plotted

$|\vec{r}_{ij}| < R_d$

$\frac{\vec{v}_i \cdot \vec{v}_j}{|\vec{v}_i||\vec{v}_j|} > \cos(A_d)$

Clusters containing more than two bacteria are plotted

$N_{total} = 343$
Collective Motion at High Density

- Clusters with a range of sizes
  - Dynamic and long-lived
  - Strong velocity correlation
- No inter-cluster correlation
  - No mean flow
  - No long-range order

Qualitatively similar to numerical simulations:
Hernandez-Ortiz et al. 2005
Saintillan & Shelley 2007
Ishikawa & Pedley 2008

Global Analysis:
Zhang et al. PNAS (2010)
Size Distributions of Bacterial Clusters

Similar distributions found in fish schools & herds:

Bonabeau et al. 1999
Couzin and Krause 2003
Peruani et al. 2012

\[ P(n) = An^{-b}e^{-n/n_c} \]
\[ A = 0.5 \]
\[ b = 1.85 \]
\[ n_c = 6.5 \text{ for } N_{\text{total}} = 343 \]
\[ n_c = 75 \text{ for } N_{\text{total}} = 718 \]
Bacteria in Large Clusters Move Faster

\[ N_{\text{total}} = 343 \quad N_{\text{total}} = 539 \quad N_{\text{total}} = 718 \]

Mean Speed (\( \mu m/s \))

\[ P(v_X) \]

Increasing size
Large Density Fluctuations

Mean:
\[ N(L = 90 \, \mu m) = 718 \]

Standard deviation:
\[ \Delta N(L = 90 \, \mu m) = 70 \]

How does \( \Delta N(L) \) Scale with \( N(L) \) ?
Anomalous Scaling in Density Fluctuations

Thermally equilibrated systems:

\[ \Delta N \propto N^{0.5} \]

Our system:

\[ \Delta N \propto N^{0.75} \]

→ Non-equilibrium systems are different.

→ Originated from active motion:

Exponent 0.8 in polar systems: observed in numerical model by Chate et al 2008; in granular system by Deseigne et al 2010;

Exponent 1 in apolar systems: predicted by Ramaswamy, Simha, and Toner 2003; observed in granular system Narayan et al 2008.
Instantaneous Configurations of Clusters

Mean velocity
\[ \vec{V}_I = \langle \vec{v}_{i,I} \rangle_i \]
Mean speed
\[ S_I = \langle |\vec{v}_{i,I}| \rangle_i \]
Polarization
\[ \vec{P}_I = \left\langle \frac{\vec{v}_{i,I}}{|\vec{v}_{i,I}|} \right\rangle_i \]
Mean orientation
\[ \vec{\Theta}_I = \left\langle \vec{\theta}_{i,I} \right\rangle_i \]

Chen, Dong, Be'er, Swinney, and Zhang, PRL, 2012
Correlation Length and Clusters Size

Correlation function:

\[ C^u(r) = \frac{1}{C^u_0} \sum_{ij} (\hat{u}_{i,I} \cdot \hat{u}_{j,I}) \delta(r - |\vec{r}_{i,I} - \vec{r}_{j,I}|) \]

Correlation length:

\[ C(r = \xi) = 0 \]

Spatial size of a cluster: \textit{maximum} distance between two bacteria
Correlation Length Scales with Cluster Size

Color: 6 global densities;
Symbols: 4 correlation functions:

\[ \xi = 0.3L \]
Scale-invariant Correlations

Rescale x axis

Line: stretched exponential fit
Scale-invariant Correlations May be Universal

Cavagna et al: experimental results from bird flocks (PNAS 2010, 2012)

Birds are different from bacteria:
→ Six orders of magnitude larger;
→ More complicated biological function;
→ Motion in 3D;
→ Flocks are more long-lived.
Correlation and Response Near Criticality: Ising Model on a 2D Square Lattice

Hamiltonian:
\[ H = -\frac{1}{2} \sum_{ij} J_{ij} s_i s_j \]

Critical temperature:
\[ T_c = \frac{2}{\ln(1+\sqrt{2})} \frac{J}{k_b} \]

Correlation length:
\[ \xi \sim \left| \frac{T-T_c}{T_c} \right|^{-1} \]

Susceptibility:
\[ \chi \sim \left| \frac{T-T_c}{T_c} \right|^{-7/4} \]

Kenneth Wilson
*Problems in physics with many scales of length*

Self-similarity ➔
Renormalization group theory
Go With Neighbors: a “Ferromagnetic” Flock

Update position:

\[ r_i^{t+\Delta t} = r_i^t + v_0 \Delta t v_i^{t+\Delta t} \]

Update velocity:

\[ v_i^{t+\Delta t} = \Theta \circ \left[ \alpha \sum_{k \in N_i} v_k^t + \beta \sum_{k \in N_i} f_{ik} e_{ik}^t + \gamma n_i \right] \]

Fixed parameters:

→ Number of neighbors = 20;
→ Number of birds 500-500000;
→ Vector noise;
→ Random initial condition;
→ Open boundaries;
→ \( \beta = 1 \);

Polarization Phase Diagram

Number of birds 500

Ordered

Dis-ordered

\[ \log_2 (\alpha/0.001) \]

\[ \log_2 (\gamma/0.001) \]
Typical Evolution of the Model

Alignment strength:
\[ \alpha = 0.001 \times 2^{10} \]

Noise strength:
\[ \gamma = 0.001 \times 2^{6} \]

Final state:
ordered state with fluctuations
Typical Evolution of the Fluctuations

Alignment strength:
\[ \alpha = 0.001 \times 2^{10} \]

Noise strength:
\[ \gamma = 0.001 \times 2^6 \]

Arrows:
Velocity fluctuations X 30
Correlation Length Scales with Cluster Size
Scale-invariant Correlations

Correlation Function vs Normalized Separation
Spatial Correlation of Fluctuations

Long-ranged correlation in all ordered states.
Concluding Remarks

Collective motion is a widely observed phenomenon:
- wide range of system sizes;
- short range interactions without central control;
- intrinsic noises cause fluctuations;
- novel statistical system far from thermal equilibrium:
  \[ P(n) = A n^{-b} e^{-n/n_c} \quad \Delta N \propto N^{0.75} \]

Scale-invariant correlations of fluctuations:
- experimental and numerical studies lead to qualitatively similar results over a wide range of parameters;
- correlation lengths proportional to cluster sizes
  \[ \xi = sL; \quad s \in [0.3, 0.4] \]
- correlation functions collapse after rescaling;

Origin: spatial self-similarity;
Function: poised at criticality to achieve sensitive response.