Synchronization in Real Optical Networks
Chaos, Communication and Chimeras in the Laboratory

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Outline

dynamics on an interacting complex network of optoelectronic chaotic nodes

application I  chaotic communication

application II  optimal network configurations

Group Synch and Chimeras
CO-AUTHORSHIP NETWORK

\[
\frac{d\mathbf{x}_i(t)}{dt} = \mathbf{F}_i \left[ \mathbf{x}_i(t) \right] + \varepsilon_i \sum_{j=1}^{N} A_{ij} \mathbf{H}_j \left[ \mathbf{x}_j(t - \tau_{ij}) \right]
\]
- What is the **dynamics** of a single node?
- How does the **network structure** $A_{ij}(t)$ influence synchronization $x_i(t)$?
- Do certain network configurations promote group synch?
- Chimera states?
Study the dynamics of a network of nominally identical *time-delayed chaotic oscillators* with experiments & an accurate *numerical model*.
optoelectronic chaos

laser diode

optical modulator

fiber optic cable

photodetector

electronic amplifier

data in

data out
optoelectronic chaos

\[ P = P_0 \cos^2 \left[ \frac{\pi V}{2V_\pi} + \phi_0 \right] \]

- \(~10 \text{ cm}\)

\[ P = P_0 \cos^2 \left[ \frac{\pi V}{2V_\pi} + \phi_0 \right] \]
optoelectronic chaos

\[ V(t) \]

Optoelectronic Chaos

$$x(t) = \frac{\pi V(t)}{2V_\pi}$$

Experiment

Simulation

$$\beta = \frac{\pi RG P_0}{2V_\pi}$$

optoelectronic chaos

\[
\frac{du(t)}{dt} = Au(t) - B \beta \cos^2 \left( x(t - \tau) + \phi_0 \right)
\]

\[x(t) = Cu(t)\]
optoelectronic chaos

ref: T. E. Murphy et al., Phil. Trans. R. Soc. A 368, 343 (2010).
optoelectronic chaos
synchronization

“ODD KIND OF SYMPATHY”
- Christiaan Huygens (1665)

\[
\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N} \sum_{j=1}^{N} \sin(\theta_j - \theta_i)
\]

Kuramoto model (1975)
Hidden messages
Injecting a message into the transmitter laser “folds” the data into the chaotic fluctuations. The receiver reverses this process, thereby recovering a high-fidelity copy of the message.

EDFA: erbium-doped fiber amplifier.

Lou Pecora, Physics World, 1998
Transmitter and receiver systems

Optoelectronic systems

a Input message

Emitter

Output message

Receiver

Claudio Mirasso
Leader of the EU consortium
Spotted here in Guanajuato, Mexico (2005)
The History of Camouflage

The term **camouflage** comes from the French word *camoufler* meaning "to blind or veil." Camouflage, also called protective concealment, means to disguise an object, in plain sight, in order to conceal it from something or someone.

In the late 1800s, an American artist named Abbott Thayer made an important observation about animals in nature that became a useful tool in developing modern camouflage. After studying wildlife, Thayer noticed that the coloring of many animals graduated from dark, on their backs, to almost white on their bellies. This is an important property that is very useful in modern camouflage. This gradation from dark to light breaks up the surface of an object and makes it harder to see the object as one thing. The object loses its three-dimensional qualities and appears flat. This tendency to break up and flatten the surface of an object also appears in the artistic movement, Cubism, which was occurring during this same time period. Camouflage, as we know it today, was born in 1915 when the French army created a new unit called the camouflage division. Artists were among the first people the French army called in to help develop camouflage for use during WWI.

after Picasso's
Seated Woman, 1909
Hedy Lamarr
MUNITIONS INVENTOR
Frequency Hopping Spread Spectrum

Hedy Kiesler Markey & George Antheil
Message communication at 1 Gbit/s

10 Gbits/s, Bit Error Rates ~ $10^{-12}$ with error correction techniques
directed graph

adjacency matrix

\[ A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ A_{N1} & \cdots & A_{NN} \end{bmatrix} \]

coupled equations of motion

\[ \frac{d\mathbf{x}_i(t)}{dt} = \mathbf{F}[\mathbf{x}_i(t)] + \varepsilon \sum_{j=1}^{N} A_{ij} \mathbf{H}[\mathbf{x}_j(t - \tau)] \]
**chaotic synchronization**

**synced network**

\[ x_1(t) = x_2(t) = \cdots = x_N(t) = s(t) \]

**synced equations**

\[
\frac{dx_i(t)}{dt} = F[x_i(t)] + \varepsilon \sum_{j=1}^{N} A_{ij} H[x_j(t - \tau)]
\]

\[
\therefore k_1 = k_2 = \cdots = k_N
\]
unidirectional coupling, \( N = 2 \)

\[ A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \]

\( \Lambda = \{1, 0\} \)


bidirectional coupling, \( N = 2 \)

\[
A = \begin{bmatrix} 1 - \kappa & \kappa \\ \kappa & 1 - \kappa \end{bmatrix}
\]

\[ \Lambda = \{1, 1 - 2 \kappa\} \]

\( \kappa = 0.2 \)

\( x_1(t) \neq x_2(t) \)

\( \kappa = 0.3 \)

\( x_1(t) = x_2(t) \)

**CHAOTIC SYNCHRONIZATION**

**synced network**

\[ x_1(t) = x_2(t) = \cdots = x_N(t) = s(t) \]

**synced equations: diffusive coupling**

\[
\frac{dx_i(t)}{dt} = F[x_i(t)] + \bar{\epsilon} \left\{ \sum_{j \neq i} A_{ij} H[x_j(t - \tau)] - k_i H[x_i(t - \tau)] \right\}
\]

\[
= F[x_i(t)] - \bar{\epsilon} \sum_{j=1}^{N} L_{ij} H[x_j(t - \tau)]
\]

\[ k_i := \sum_{j=1}^{N} A_{ij} \]
Q: can we determine how well a particular topology of chaotic oscillators will synchronize?

SYNCHRONIZABILITY

- over what range of coupling strengths $\varepsilon$ does the network exhibit synchrony?
- what is the coupling cost $\varepsilon_{\text{min}}$ for the network to synchronize?
- how fast ($e^{-\mu t}$) does the network converge to a synchronous state?
Q: can we determine how well a particular topology of chaotic oscillators will synchronize?

A: yes, the variance of eigenvalues $\sigma$ of $L$ is a measure of these synchronization properties:

$$\sigma^2 = \frac{1}{d^2(N-1)} \sum_{i=2}^{N} \left| \lambda_i - \bar{\lambda} \right|^2$$

where

$$d := \frac{m}{N} = \frac{\text{number of links}}{\text{number of nodes}}$$

networks that minimize $\sigma$ have optimal synchronization properties.

binary directional coupling, $N = 4$

A quantized number of links $m = q (N - 1)$ minimizes variance $\sigma$, $q = 1, 2, \ldots, N$
**Application II**

**Optimal Networks**

\[
\text{sync. error} = \theta(t) = \frac{1}{N(N-1)} \sum_{i,j} |x_i(t) - x_j(t)| \sim e^{-\mu t}
\]
Four-node network:
application II

optimal networks

less can be more!

Influence of network structure:

Does the synchronizability measure $\sigma$ completely characterize the synchronization properties?

For co-spectral configurations with $m$ links, does the connection geometry matter?

\[
L^{(n)} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 \\
-1 & 0 & 1 & 0 \\
-1 & 0 & 0 & 1
\end{pmatrix}
\]

\[\Lambda = \{0,1,1,1\}\]

\[
L^{(s)} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 \\
0 & -1 & 1 & 0 \\
0 & 0 & -1 & 1
\end{pmatrix}
\]

\[\Lambda = \{0,1,1,1\}\]

*diagonalizable* (linearly independent eigenvectors)  
*nondiagonalizable* (linearly-dependent eigenvectors)
Influence of network structure:

- *nondiagonalizable* configurations have an initial polynomial convergence transient.

\[ \theta(t) \equiv \frac{1}{N(N-1)} \sum_{i,j} |x_i(t) - x_j(t)|. \]

Experiment

Simulation
Influence of network structure:

- nondiagonalizable configurations also exhibit greater variability among different realizations and are in general more sensitive to perturbations. Hence we term these configurations as *sensitive*.

Ring of coupled oscillators

Time Traces for Different Coupling Delays

As the coupling delay changes, we observe one of four distinct synchronization states between the four coupled oscillators:

\[ \tau_c = \tau_f \]
Isochronal Synchrony
(Phase = 0)

\[ \tau_c = 1.26 \tau_f \]
Splay-phase Synchrony
(Phase = \( \pi/2 \))

\[ \tau_c = 1.53 \tau_f \]
Cluster Synchrony
(Phase = \( \pi \))

\[ \tau_c = 1.79 \tau_f \]
Splay-phase Synchrony
(Phase = \( 3 \pi/2 \))

At intermediate coupling delays, bistability is observed between pairs of these states.
Phase relationships of synchrony

We performed experiments and simulations over a range of coupling delays. For each coupling delay, the system is started from random initial conditions. For the different initial conditions, we record how many result in each of the four different behaviors.

Experiment
(10 measurements per time delay)

Simulation
(2000 random initial conditions per time delay)
Group Synch Experiments

work in progress – Williams, Dahms, Schöll, Sorrentino

Nodes 1 and 3: identical feedback strength
Nodes 2 and 4: identical (but different) feedback strength

The two sets of nodes have different chaotic attractors
Group Synch Experiment

Uncoupled Nodes

Coupled Nodes

- Amplitude (A.U.)
- Time (ms)

Node 1, Node 2, Node 3, Node 4
Chimeras

Kuramoto, Battoghtokh 2002

Abrams, Strogatz PRL 2004
The Chimera is any mythical animal with parts taken from various animals and, more generally, an impossible or foolish fantasy (Wiki)

Sync and desync coexist in ensembles of non-locally coupled, identical nonlinear dynamical phase oscillators
Can they be observed in real experimental systems?
Discussions with Y. Kuramoto, S. Strogatz, Carlo Laing, ............K. Showalter!

Loss of Coherence in Dynamical Networks: Spatial Chaos and Chimera States

Iryna Omelchenko,¹,² Yuri Maistrenko,²,³ Philipp Hövel,¹ and Eckehard Schöll¹

\[ z_{i+1}^{t} = f(z_{i}^{t}) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} [f(z_{j}^{t}) - f(z_{i}^{t})], \]

\[ f(z) = az(1-z), \quad a = 3.8 \]
Nonlocally coupled chaotic logistic maps
(Also, coupled Rössler oscillators)
Omelchenko et al., PRL

FIG. 1 (color online). Regions of coherence for system (1) in the \((r, \sigma)\) parameter plane with wave numbers \(k = 1, 2,\) and \(3\). Snapshots of typical coherent states \(z_i\) are shown in the insets. The color code inside the regions distinguishes different time periods of the states. The coherence-incoherence bifurcation (CIB) curve separates regions with coherent and incoherent dynamics. In the hatched and shaded (color) regions below CIB, two-cluster incoherent states exist. Completely synchronized chaotic states exist in the light hatched region bounded by the blowout bifurcation curve BB. Parameters: \(a = 3.8\) and \(N = 100\).
Chaotic Iterated Map

\[ \phi_{t+1} = 0.85 \pi [1 - \cos(\phi_t)] \]

- Can be implemented experimentally
Bellerophon on Pegasus spears the Chimera, 425–420 BC
These types of systems can be experimented on in different world locations – in Beijing.........
Spatial Light Modulator

- http://www.bnonlinear.com/
How to Make Maps

- 0) Partition the SLM screen into regions
- 1) Record intensity with camera
- 2) Couple regions with computer
- 3) Update phase shifts applied by SLM
- 4) Go to 1

\[
\phi_{i}^{t+1} = 2\pi a \left\{ I(\phi_{i}^{n}) + \frac{\varepsilon}{2R} \sum_{j=-R}^{R} [I(\phi_{j+i}^{t}) - I(\phi_{i}^{t})] \right\}
\]

- Intensity measured by camera
- Scaling (by computer)
- Updated phase
- Coupling between elements (In computer)
Coupled Iterated Maps

Local dynamics

\[ \phi_{i}^{t+1} = 2\pi a \left\{ \left[ I(\phi_{i}^{n}) \right] + \frac{\varepsilon}{2R} \sum_{j=-R}^{R} \left[ I(\phi_{j+i}^{t}) - I(\phi_{i}^{t}) \right] \right\} \]

Diffusive coupling

\[ I(\phi) = \frac{1}{2} (1 - \cos \phi) \]

Periodic Boundary Conditions

• Implemented experimentally!
1 D Parameter space
Three representative examples showing intensity as a function of position in the 1d system.

At high coupling strengths, the patterns are coherent (A), and at low coupling strengths they are incoherent (C).

Intermediate coupling strengths have coexisting domains of coherence and incoherence (B). Chimera States
2d System

\[ \phi_{i,j}^{t+1} = 2\pi a \left\{ I(\phi_{i,j}^t) + \frac{\varepsilon}{4R^2} \sum_{m=-R}^{R} \sum_{n=-R}^{R} \left[ I(\phi_{j+m,k+n}^t) - I(\phi_{i,j}^t) \right] \right\} \]

\begin{tabular}{cccccccc}
\( \varphi_{00} \) & \( \varphi_{01} \) & \( \varphi_{02} \) & \( \varphi_{03} \) & \( \varphi_{04} \) & \( \varphi_{05} \) & \( \varphi_{06} \) & \( \varphi_{07} \) \\
\( \varphi_{10} \) & \( \varphi_{11} \) & \( \varphi_{12} \) & \( \varphi_{13} \) & \( \varphi_{14} \) & \( \varphi_{15} \) & \( \varphi_{16} \) & \( \varphi_{17} \) \\
\( \varphi_{20} \) & \( \varphi_{21} \) & \( \varphi_{22} \) & \( \varphi_{23} \) & \( \varphi_{24} \) & \( \varphi_{25} \) & \( \varphi_{26} \) & \( \varphi_{27} \) \\
\( \varphi_{30} \) & \( \varphi_{31} \) & \( \varphi_{32} \) & \( \varphi_{33} \) & \( \varphi_{34} \) & \( \varphi_{35} \) & \( \varphi_{36} \) & \( \varphi_{37} \) \\
\( \varphi_{40} \) & \( \varphi_{41} \) & \( \varphi_{42} \) & \( \varphi_{43} \) & \( \varphi_{44} \) & \( \varphi_{45} \) & \( \varphi_{46} \) & \( \varphi_{47} \) \\
\( \varphi_{50} \) & \( \varphi_{51} \) & \( \varphi_{52} \) & \( \varphi_{53} \) & \( \varphi_{54} \) & \( \varphi_{55} \) & \( \varphi_{56} \) & \( \varphi_{57} \) \\
\( \varphi_{60} \) & \( \varphi_{61} \) & \( \varphi_{62} \) & \( \varphi_{63} \) & \( \varphi_{64} \) & \( \varphi_{65} \) & \( \varphi_{66} \) & \( \varphi_{67} \) \\
\( \varphi_{70} \) & \( \varphi_{71} \) & \( \varphi_{72} \) & \( \varphi_{73} \) & \( \varphi_{74} \) & \( \varphi_{75} \) & \( \varphi_{76} \) & \( \varphi_{77} \) \\
\end{tabular}

White is coupled to yellow.
2D Parameter space

(a) 2D Parameter space plot showing different regions such as Chaos, Global Sync., and Incoherence.

(b) 3D Simulation and Experiment index plots illustrating intensity variations.
Compare 1D and 2D systems
HandsOn Research, Shanghai
Waiting to begin!
Spiral Wave Chimeras?!

Excitation threshold, refractory period, time delayed feedback, noise
Spiral Wave Chimeras

FIG. 4: Spiral pattern (left) and its core structure (right) exhibited by non-locally coupled phase oscillators governed by Eq. (11), where $\alpha = 0.3$.

Optoelectronic time-delayed feedback loops are a good testbed for studying nonlinear dynamics. Application to chaotic communication.

Networks with optimal sync properties – network topology influences convergence to synchronization.

Study synchronization in large 1d and 2d networks of dynamical nodes – liquid crystal spatial light modulators

Chimera states observed experimentally – work in progress!
ACKNOWLEDGEMENTS


Those who understand others are intelligent
Those who understand themselves are enlightened

Those who overcome others have strength
Those who overcome themselves are powerful

Those who know contentment are wealthy
Those who proceed vigorously have willpower

Those who do not lose their base endure
Those who die but do not perish have longevity

Chapter 33, Tao Te Ching