Bose Einstein Condensation

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Bose Einstein Condensates(BEC)

- Fifth state of Matter
- Predicted by Bose and Einstein in 1924
- Experimental realization - By Eric Cornell, Carl Wieman in Rubidium atoms at 1995 - Wolfgang Ketterle created BEC with Sodium atoms
- Received Nobel Prize in Physics in 2001

Mathematical Model : GP equation

At ultra low temperature the dynamics of BECs can be described by the three dimensional GP equation of the form

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + g_0|\psi(\vec{r},t)|^2 + V(\vec{r},t)\right)\psi(\vec{r},t) \tag{1}$$

where $\psi(\vec{r},t)$ represents the condensate wave function. $V(\vec{r})$ represents trapping potential to be $V(\vec{r})=m(\omega_r^2 r^2+\omega_z^2 z^2)$, $\omega_{r,z}^2$ are the confinement frequencies in the radial and axial directions respectively. Interatomic strength $g_0=4\pi\hbar^2 a/m$, a is scattering length, m is the atom mass.

Solving Two Dimensional coupled GP equation

We investigate the (2+1) dimensional coupled GP equation with an arbitrary time dependent tapping potential of the form

$$i\frac{\partial\psi_j}{\partial t} = -\frac{1}{2}\Big(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\Big)\psi_j + g(|\psi_1|^2+|\psi_2|^2)\psi_j + \frac{1}{2}\Omega(t)^2(x^2+y^2)\psi_j = 0 \tag{2}$$

where j=1,2. To solve eq.(2), we use Hirota method. To generate Hirota bilinear form, we use the Hirota bilinear transformation in the form

$$\psi_j = e^{i\alpha(t)((x^2+y^2)/2)}\frac{G^j}{F} \tag{3}$$

where $G=G(x,y,t)$ is complex function, $F=F(x,y,t)$ is real function. Substituting eq.(3) into eq.(2) yields

$$[iD_t = \frac{1}{2}(D_x^2+D_y^2) + i\alpha(t)xD_x + i\alpha(t)yD_y + i\alpha(t)]G^j.F = 0$$

$$\frac{1}{2}(D_x^2+D_y^2)F.F = gG^jG^{j*} \tag{4}$$

The trapping potential $\Omega(t)^2$ related with the function $\alpha(t)$ as

$$\Omega(t)^2 = \frac{d\alpha(t)}{dt} + \alpha(t)^2 \tag{5}$$

The operator D_x and D_t are defined as

$$D_t^m D_x^m(G.F) = \Big(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\Big)^m\Big(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\Big)^n G(t,x)F(t',x')|t=t', x=x' \tag{6}$$

The functions G^j and F can be extended as

$$G^j = \epsilon g_1^j + \epsilon^3 g_3^j + \epsilon^5 g_5^j + ...$$
$$F = 1 + \epsilon^2 f_2 + \epsilon^4 f_4 + ... \tag{7}$$

To generate one soliton solution we take g_1^j and f_2 as

$$g_1^j = C_1^j e^{\chi_1}$$
$$f_2 = \frac{(|C_1^1|^2+|C_1^2|^2)e^{2\int\alpha(t)dt}}{(a+a^*)^2+(b+b^*)^2}e^{\chi_1+\chi_1^*} \tag{8}$$

where

$$\chi_1 = ae^{-\int\alpha(t)dt}x + be^{-\int\alpha(t)dt}y + \int[\frac{i}{2}(a^2+b^2)e^{-2\int\alpha(t)dt} - \alpha(t)]dt$$

where a and b are complex constants. Substituting g_1^j and f_2 in eq.(2), we get the one soliton solutions of the form

$$\psi_j = A_j(a_R^2+b_R^2)^{\frac{1}{2}}Sech(\chi_{1R}+\eta/2)e^{i\chi_{Im}+i\alpha(t)(x^2+y^2)/2} \tag{9}$$

where e^{η}=g $\frac{(|C_1^1|^2+|C_1^2|^2)e^{2\int\alpha(t)dt}}{(a+a^*)^2+(b+b^*)^2}$, A_j=$\frac{C_1^j}{g(|C_1^1|^2+|C_1^2|^2)}e^{2\int\alpha(t)dt}$

From the above solution, we can observe the amplitude A, position $(\chi_{1R}+\eta/2)$ of the soliton depends on the trapping potential. While modulating the trapping potential, one can easily control the physical properties of the condensates.

References

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