The Institute for Research and Community Services
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Preventing Wax Formation in an Oil Flow in a Deepwater Riser

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1. Introduction

An oil flowing in a deep offshore riser will experience a considerable drop in temperature due to temperature difference with surrounding. The temperature of deep sea water is ranging between 2-4°C at sea bed. This can disturb the flow since wax may be formed. The situation should be avoided by reducing heat loss from the flow in the riser into the surrounding environment by providing an insulator on the riser. In this study, the location and specification of insulator are determined based on a mathematical model of the heat loss.

The heat loss of oil flow in the riser depends on the flow condition (temperature, flow rate, gas physical properties, thermal properties of gases and riser, etc.), and environmental condition of seawater around the riser (temperature, ocean currents, the physical and thermal properties of seawater). Seawater flow conditions around the riser can be divided into three zones of flow, which are static zone where no flow occurs, laminar flow zone, and turbulent flow zone near the ocean surface. This research is also intended to analyze the effect of laminar flow of seawater to the oil flow.

2. Objective

- Preventing wax formation of the oil flow in a deepwater riser by considering the effect of laminar flow of seawater. This practically means that the location and specification of insulator have to be predicted.
- Developing software based on the study.

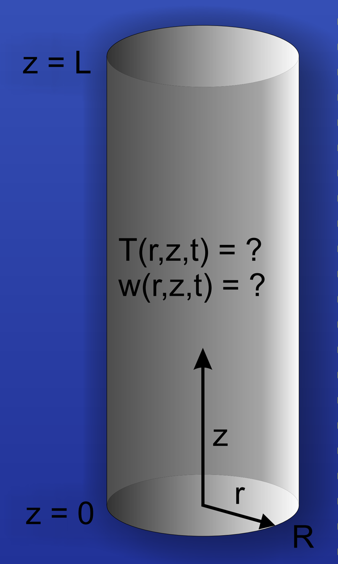
3. Methodology

A 2-D mathematical model describing oil flow in a riser is formulated. It consists of Navier-Stokes Equation and the equation of energy conservation forming a system of PDEs. The effect of the sea water to the system of interest is represented by a boundary condition at pipe wall having a term of surrounding temperature. The temperature of sea water is modeled generally to cover the existence of dynamic laminar flow. The mathematical model is solved analytically, and numerically by Finite Difference Methods.

4. Mathematical Model

Assumptions

1. The fluid flowing is assumed to be an incompressible Newtonian liquid
2. The liquid flows in vertical direction only but temperature varies both in radial and vertical direction
3. Friction factor between fluid and pipe wall is neglected
4. The vertical velocity does not depend on time, it is only a function of radial direction (radially symmetric flow)
5. The temperature surrounding the pipe (sea temperature) linearly decreases with respect to the depth



Variables	Description
T(r,z,t)	Temperature
w(r,z,t)	Velocity

Parameters	Description	Value	Unit
L	pipe length	1000	m
R	pipe radius	0.1016	m
ρ	fluid density	880	kg/m³
g	gravity acceleration	9.8	m/s²
μ_R	fluid viscosity	0.02	Pa.s
κ	thermal diffusivity	0.13	joule/(s.m.°K)
C_p	fluid heat capacity	1528	joule/(kg.°K)
ΔP	pressure difference	8659433	Pa

Governing Equations

Fluid incompressibility

$$\nabla \cdot \vec{v} = 0 \quad \text{where} \quad \vec{v} = \begin{bmatrix} u(r,z,t) \\ v(r,z,t) \\ w(r,z,t) \end{bmatrix}$$

Navier-Stokes Equation

$$\rho\left[\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial z}\right] = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2}\right] - \rho g, \quad 0<r<r_R, 0<z<z_R, t>0$$

Energy Conservation

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \frac{\kappa}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right], \quad 0<r<r_R, \quad 0<z<z_R, \quad t>0$$

Dimensionless Form

(with small parameter $\varepsilon = \frac{r_R}{z_R} << 1$)

1. Navier-Stokes Equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) = \mathrm{Re}\left[\frac{P_R \varepsilon}{\rho w_R^2}\frac{\partial P}{\partial z} + \frac{g r_R}{w_R^2}\right], \quad 0<r<1, \ 0<z<1, \ t>0$$

Boundary Conditions

$$\frac{\partial w(0)}{\partial r} = 0, \quad w(1) = 0$$

Analytical Solution

$$w(r) = \frac{1}{4}\left[\frac{\partial P}{\partial z} - 1\right](1-r^2)$$

2. Energy Conservation

$$\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z} = \frac{\kappa z_R}{\rho C_p w_R r_R^2}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right], \quad 0<r<1, \ 0<z<1, \ t>0$$

Initial Condition

$$T(r,z,0) = \frac{T_0}{T_R}$$

Boundary Conditions

$$\frac{\partial T(0,z,t)}{\partial r} = 0$$

$$\frac{\partial T(r_R,z,t)}{\partial t} = -\frac{r_R}{\kappa} h\left(T(r_R,z,t) - T_S(z)\right)$$

$$T(r,0,t) = 1$$

with $h = \frac{h_p}{t_h}$

Surrounding Temperature

To cover the effect of laminar flow of sea water, the surrounding temperature is modeled as

$$T_s(z,t,v_s) = a z + T_s(z=0) - b v_s(z,t)$$

with dynamic laminar flow

$$v_s(z,t) = \begin{cases} k_1 + k_2 \sin(k_3 t), & z_1 \le z \le z_2 \\ 0, & otherwise \end{cases}$$

Notations

- T_R reference temperature
- w_R reference velocity
- P_R reference pressure
- z_R reference length
- r_R reference radius
- h_p pipe conductivity
- t_h pipe wall thickness
- Re Reynold's number
- T_s surrounding temperature
- v_s surrounding velocity
- a gradien constant
- b, k_i velocity constants

5. Numerical Simulation

The system of equations above is solved numerically by Finite Difference Method (FDM). The velocity profile (Fig. 1) along the radial direction is calculated directly from the analytical solution.

The temperature profile is simulated in transient case. Forward difference is applied to time derivative and upwind scheme is applied to convective term. Special treatment should be given for radially symmetric boundary condition since the cylindrical coordinate is considered.

The parabolic profile of velocity, in Fig.1, along radial direction is obviously obtain from the Poisson equation with no-slip boundary condition at pipe wall and symmetric boundary condition at its center. This shape does not vary in depth and time.

At particular depth the temperature varies (Fig. 2) in time and space (radius). Once the convective term reaches this position, the temperature at the center of pipe will be increasing gradually in time. Sea water produces the depression of the curve at pipe wall.

Fig. 3 describes the two dimensional temperature distribution of fluid for different time steps. Along z direction the fluid flows with certain temperature from the well head. Meanwhile, cooling phenomenon occurs do to temperature difference between fluid and surrounding.

Practically the surrounding temperature can possibly cause a serious problem to the flow. For instance, wax might be formed when the critical temperature is achieved. This case is illustrated in Fig. 4. To overcome this condition we can set up the material (pipe) parameter. To avoid the problem we have to decrease the pipe conductivity by installing an insulator, or setting the pipe material and its thickness. This case is shown by Fig. 4 (right).

The existence of dynamic laminar flow of sea water will increase the heat loss. Therefore, the formation of wax might also occur at this region, as can be seen in Fig. 5.

6. Result

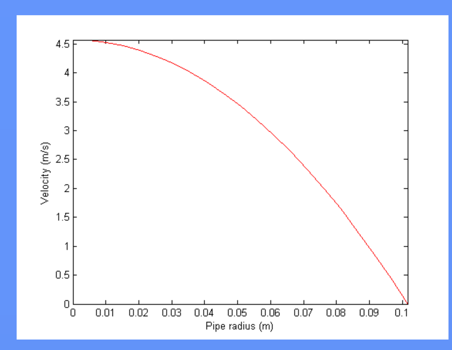


Fig. 1 Velocity of fluid in radial direction

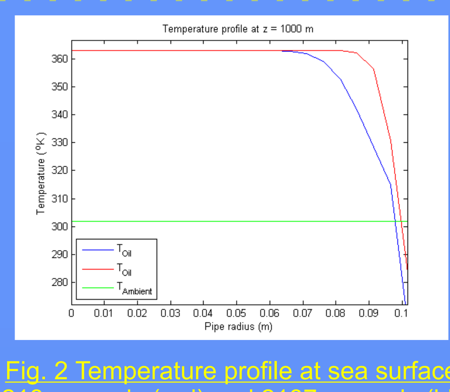


Fig. 2 Temperature profile at sea surface at 218 seconds (red) and 2187 seconds (blue)

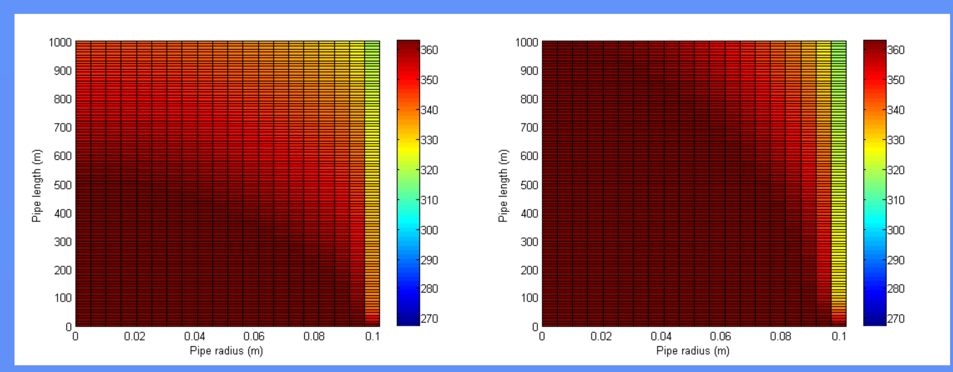


Fig. 3 Temperature profile at different times: after 108 (left) and 217 (right) seconds

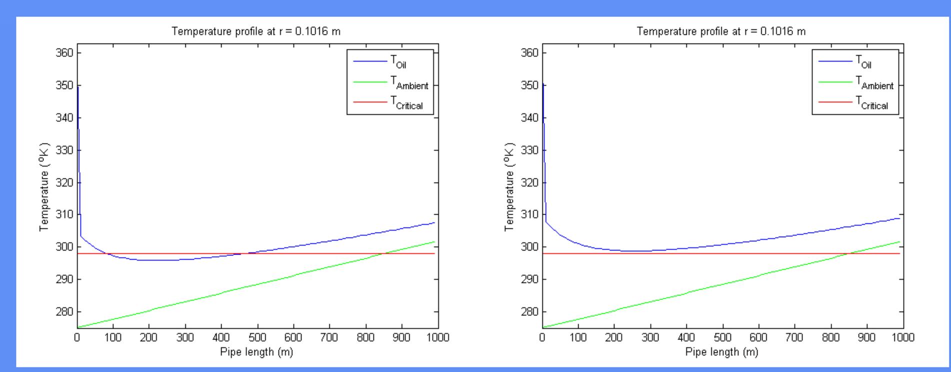


Fig. 4 Temperature profile at pipe wall for h = 50 (left) and h = 40 (right)

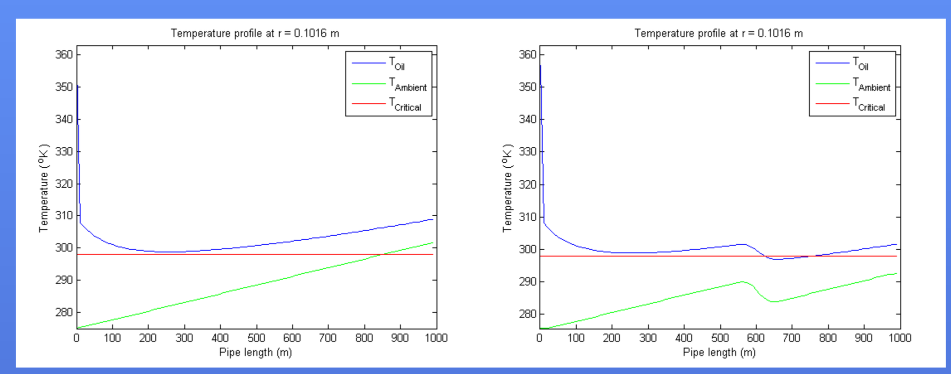


Fig. 5 Temperature profile at pipe wall for h = 40 without (left) and with (right) sea current

7. Conclusion

- Distribution of an oil temperature in a riser is mainly depend on material properties (pipe conductivity and wall thickness), fluid properties (Reynold Number), and surrounding temperature (sea temperature).
- To prevent wax formation, the ratio between pipe conductivity and wall thickness has to be set appropriately.
- Practically, heat loss can be reduced by installing insulator at the position that is predicted by the present model.

8. Software

To predict location and specification of insulator, a software is being developed using C++ after its prototype is tested.

9. Acknowledgement

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