Dynamics of β-Pasta Ulam potential in 1 dimensional N particle leads to determine non-linear equation of motion. For nearest neighbouring interaction we can categorize the equation of motion which leads to periodical N particle leads to discrete non-linear equation.

SIMPLE PERIODIC ORBITS

For an N-particle system, enumeration of periodic orbits is a difficult task. In the following, we restrict ourselves to what can be termed as ‘Simple periodic orbits’ as all the individual particles satisfy the same equation:

\[ x_j = f(x_j) \]

for any J. We enumerate possible phase-relationships of particles on a 1-D dPPU lattice such that the motion of all the particles is governed by a single equation of motion. As we consider only nearest-neighbour interaction, we group these basic arrangements of triplets (j)th, (j+1)th and (j+2)th particles that yield periodic solution on repetition. Six such arrangements are noted below.

**Case I:**

\[ j \]

\[ \theta_1 - \theta_2 - \theta_3 = 0 \]

\[ x_j = 0 \]

for \( j = 1, 2, \ldots, N \).

**Case II:**

\[ j \]

\[ \theta_1 - \theta_2 - \theta_3 = \pi \]

\[ x_j = \pm \theta_3 \]

for \( j = 1, 2, \ldots, N/2 \) and \( N = 4n \) where \( n = 1, 2, \ldots \).

**Case III:**

\[ j \]

\[ \theta_1 - \theta_2 - \theta_3 = 2\pi \]

\[ x_j = \pm 2\theta_3 \]

for \( j = 1, 2, \ldots, N/2 \) and \( N = 4n \) where \( n = 1, 2, \ldots \).

**Case IV:**

\[ j \]

\[ \theta_1 - \theta_2 - \theta_3 = 3\pi \]

\[ x_j = \pm 3\theta_3 \]

for \( j = 1, 2, \ldots, N/2 \) and \( N = 3n \) where \( n = 1, 2, \ldots \).

**Case V:**

\[ j \]

\[ \theta_1 - \theta_2 - \theta_3 = \pm 4\pi \]

\[ x_j = \pm 4\theta_3 \]

for \( j = 1, 2, \ldots, N/2 \) and \( N = 4n \) where \( n = 1, 2, \ldots \).

**Case VI:**

\[ j \]

\[ \theta_1 - \theta_2 - \theta_3 = \pm 5\pi \]

\[ x_j = \pm 5\theta_3 \]

for \( j = 1, 2, \ldots, N/2 \) and \( N = 3n \) where \( n = 1, 2, \ldots \).

For each distinct eigenvalue, we solve the eigenvalue equation to find eigenvalues \( x_j(\tilde{\gamma}) \). Using the solution (11) and defining \( n = N \), we get the Lamé equation for \( x_j(u) \):

\[ x_j(u) + Q(x_j(u)) = 0 \]

The function \( x_j(u) \) is a linear combination of \( x_j(\tilde{\gamma}) \) and \( \tilde{\gamma}(\gamma) = \frac{dx_j(u)}{du} \) for each distinct eigenvalue. The function \( Q(u) \) takes distinct forms for each eigenvalue. According to the Floquet theory \( Q(x) \) is a periodic function. For Case II, \( Q(x) = 2\tilde{x} \cos(\tilde{x}) \).

\[ Q(x) = 2\tilde{x} \cos(\tilde{x}) \]

The stability of the solution is determined by the boundedness of \( x_j(u) \). We express \( Q(u) \) and \( x_j(u) \) in Fourier series:

\[ Q(u) = \sum_{n=-\infty}^{\infty} a_n \exp(i\alpha n \pi) \]

\[ x_j(u) = \sum_{n=-\infty}^{\infty} b_n \exp(i\alpha n \pi) \exp(\gamma u) \]

where \( \gamma \) is the Floquet exponent.