Abstract:

Granular materials, collections of particles which dissipate energy through inter-particle interactions, are important in a wide variety of industries such as energy production, food processing, and pharmaceuticals. A fundamental understanding of granular systems, comparable to our current understanding of fluids and solids, does not exist today but would have far reaching implications across many industries.

Like ordinary materials, granular systems can act like a fluid or solid depending on the granular temperature, defined as the average kinetic energy of the particles. Unlike ordinary materials, granular systems can change between solid to fluid (freeze/melt) over small intervals of space or time, due to granular temperature that often vary by 4 orders of magnitude or more in the same system. Further, granular system can be melted by small strains and often flow like supersonic fluids.

These properties explain many of the fascinating flow structures seen in granular systems. One of the most intriguing and beautiful is the patterns formed in vertically vibrated thin layers. Squares, stripes, hexagons, kinks, phase domains, and solitary structures can form, depending on the frequency and strength of shaking. These patterns states allow us to test microscopic law of granular interactions using molecular dynamics. Because granular systems often flow like supersonic fluids, shocks can easily be form when a granular stream hits a stationary obstacle. This type of system allows us to test continuum theories of granular flows like Navier-Stokes for ordinary fluids. Finally, rotating drum flows elucidates the difficulties that lie ahead for a unified granular continuum theory, displaying in one flow, solid, fluid, melting, freezing, super- and sub-sonic behavior.
Problems in Granular Materials

Mark D. Shattuck
Rohit Ingale
Pedro Reis
Benjamin Levich Institute
City College of New York
NSF-DMR

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Granular Applications

Sand dune formation

Food grains

Pharmaceutical Industry

Chemical Industry
Granular Applications

Planetary rings

Space Exploration

NASA
Mars Rover
Experimental Setup

Camera

- Oscillates Sinusoidally
  \[ y(t) = A \sin(2\pi ft) \]
- Two Control Parameters
  - Max. Acceleration: \( \Gamma = \frac{A(2\pi f)^2}{g} \)
  - Frequency: \( f' = \frac{f\sqrt{g/H}}{f} \)
Patterns in Vibrated Granular Media
Patterns in Vibrated Granular Media
Square Patterns in Vibrated Granular Media
Squares, Stripes, Hexagons

(a) \( \Gamma = 3.00 \)  \( f^* = 0.27 \)
(b) \( \Gamma = 3.00 \)  \( f^* = 0.44 \)
(c) \( \Gamma = 4.00 \)  \( f^* = 0.38 \)
(d) \( \Gamma = 4.00 \)  \( f^* = 0.38 \)

Bizon, MDS, McCormick, Swift, Swinney
PRL 1998
Phase Diagram for Vibrated Granular Media

Moon, MDS, Bizon, Goldman, Swift, Swinney PRE 2002
Phase Diagram for Vibrated Granular Media

Moon, MDS, Bizon, Goldman, Swift, Swinney PRE 2002
Phase Diagram for Vibrated Granular Media

Moon, MDS, Bizon, Goldman, Swift, Swinney PRE 2002
Phase Bubbles

Moon, MDS, Bizon, Goldman, Swift, Swinney PRE 2002
Phase Diagram for Vibrated Granular Media

Moon, MDS, Bizon, Goldman, Swift, Swinney PRE 2002
Stripe Patterns in Vibrated Granular Media
Targets, Spirals, and Chaos

de Bruyn, Lewis, MDS, Swinney PRE 2001
Phase Diagram for Vibrated Granular Media

Moon, MDS, Bizon, Goldman, Swift, Swinney PRE 2002
Emergence of Order and Pattern Coarsening

Goldman, MDS, Swinney, Gunaratne Physica A 2002
Square Lattice Dynamics
Pattern Crystal with “Phonons”

Goldman, MDS, Moon, Swift, Swinney PRL 2003
Phase Diagram for Vibrated Granular Media

Moon, MDS, Bizon, Goldman, Swift, Swinney PRE 2002
Localized Patterns in Vibrated Granular Media (Oscillons)
Granular Fluid Dynamics

Mass:
\[ \frac{\partial n}{\partial t} = \nabla \cdot n \mathbf{u} \]

Momentum:
\[ n \frac{D \mathbf{u}}{D t} = -\nabla \cdot \mathbf{P} \]

Fluctuational Energy Balance:
\[ n \frac{D T}{D t} = -\kappa \nabla T + \mathbf{P} : \mathbf{E} - \gamma \]

\[ \frac{D}{D t} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \]
Dense Granular Flow
Dense Granular Flow
Microscopic Definition of Supersonic Flow

Ma$<<1$
Subsonic

Ma$>>1$
Supersonic
Supersonic flow past a wedge

Rericha, Bizon, MDS, Swinney PRL 2002
Dense Granular Flow
(Solid)
Hopper Flow
## Granular Thermodynamics

### Molecular Systems
- Conserve energy
- Equilibrium steady state (Thermodynamics)
  - Unforced.
  - Time independent.
  - Uniform.
  - Fluid-Solid phase transition.
  - Extremum principle well established (e.g. entropy and free energy).
  - State depends only on small number of variables.

### Granular Systems
- Dissipate energy
- Non-equilibrium steady state (NESS)
  - Forcing required.
  - Time independent.
  - Uniform.
  - Fluid-Solid phase transition.
  - Can we explain NESS phase transitions using free energy arguments?
  - Can we predict State from small number of variables?
Granular Thermodynamics

Isochoric Granular Fluid
Uniformly Heated (Isochoric)

Track all particles ±0.5µm

Side View:
- D=1.19mm
- 16x16 LED array
- Vertical vibration
- Rough glass bottom plate
- Flat glass bottom plate

Top View:
- Fast CCD camera
- @ 840 frames per second
- 10.2 cm
- 15 mm

Experimental parameters:
- frequency: \( f \)
- dimensionless acceleration: \( \Gamma = \frac{A(2\pi f)^2}{g} \)
- filling fraction: \( \phi \)

\[
\phi = \frac{N \pi (D/2)^2}{\pi R^2}
\]
Nonequilibrium phase transition

Granular systems
-Solid/Fluid phase transition

- Inelastic hard sphere system
- Energy dissipated
- Free energy/Entropy concepts not established

- Can we explain phase transition using thermodynamic ideas?
- Is there a functional form analogous to free energy?

Reis, Ingale, MDS PRL 96,258001 (2006)
  - Structure identical to equilibrium.

Reis, Ingale, MDS PRL 98,188301 (2007)
  - Caging dynamics.

Reis, Ingale, MDS PRE 75,051311 (2007)
  - Velocity distribution.
Caging Dynamics

- Diffusive behavior at low density.
  - Ballistic -> Diffusive
- Caging behavior at intermediate density.
  - Ballistic -> Sub-diffusive -> Diffusive
- Crystalline behavior at high densities
  - Ballistic -> No Diffusion (stuck)

Reis, Ingale, MDS PRL 98 (2007)
Caging Dynamics

- **Diffusive** (low density)
  - Ballistic (short)
  - Diffusive (long)

- **Caging** (intermediate density)
  - Ballistic (short)
  - Sub-diffusive
  - Diffusive (long)

- **Crystalline** (high densities)
  - Ballistic (short)
  - No Diffusion (long)

Reis, Ingale, MDS PRL 98 (2007)
Caging Dynamics


Identical scenario to that for MD in Q2D system of colloidal particles!
Isobaric Granular Fluid

First-order phase transition in a non-equilibrium-steady-state
Experimental setup: Isobaric

- Stainless steel ball bearings
  - N=68, D=3.175mm
- Thin (2D) container dimensions
  - Width=17.5D x Height=20D x Depth≈1D
- Constant external pressure (isobaric)
  - Floating weight on top: W/W_p ≈ 1
- Heated from below
  - Oscillating sinusoidally (f=50Hz)
    - y(t)=Asin(2πft)
- 840fps High speed digital camera
- High intensity LED light source

Fixed: P,N,Q
Free: T,V

Q(A,f)=Heat Flux
Area Fraction ν or \( \phi = NV_p/V \)
Sublimation Transition

Crystalline State

Gas State
Sublimation Transition
With Rate Dependent Hysteresis
Sublimation Transition
With Rate Dependent Hysteresis

Transition region

Volume fraction (

Maximum Acceleration (\(\Gamma = A_\infty^2 / g\) )

4.29 g/min
2.14 g/min
0.39 g/min
Volume Fluctuations

\[ \Gamma = 7.94 \]

\[ \Gamma = 8.0 \]

\[ \Gamma = 8.07 \]

Volume fraction, \( \nu \)

Time (sec)

Probability \( P(\nu) \)

Solid

Fluid

Probability \( P(\nu) \)
Functional Form of $P(\nu)$

\[ P(\nu) = Ae^{-F(\nu)} \]

\[
F(\nu) = \frac{B}{4} \left( \nu^4 - \frac{4C}{3} \nu^3 + 2D\nu^2 - 4Ev \right)
\]

\[
C = \nu_f + \nu_o + \nu_s
\]
\[
D = \nu_f \nu_s + \nu_s \nu_o + \nu_f \nu_o
\]
\[
E = \nu_f \nu_o \nu_s
\]

Analogous to Ginzburg-Landau free energy functional.

$\nu_f$ = Fluidus point
$\nu_s$ = Solidus point
$\nu_o$ = Minima location

O(4) polynomial
Results

Phenomenological Model

– System state corresponds to minimum $F(\nu)$ i.e., maximum $P(\nu)$.

– Analogous to free energy minimization in equilibrium systems.

$$P(\nu) = A e^{-F(\nu)}$$
Thermodynamic State Variable

Energy Flux? => $A\omega$

![Graph showing the relationship between $A\omega$ and the height of the layer (D)]

- 35Hz: red line (up) and blue line (down)
- 70Hz: red line (up) and green line (down)
Effect of Frequency

![Graph showing the effect of frequency on volume fraction. The x-axis represents maximum acceleration ($\Gamma = \frac{A\omega^2}{g}$), and the y-axis represents volume fraction ($\nu$). Different frequencies (20Hz, 30Hz, 40Hz, 50Hz, 60Hz, 70Hz, 90Hz) are indicated by colored lines, each showing the trend as acceleration increases.]
Collapse for High Frequencies
The End