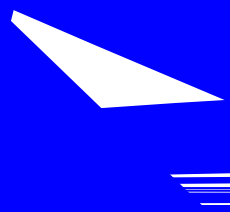


Irregular
in
Time & Space

Gravity & Turbulence
Determine
Star Formation

Turbulence & Fuel Consumption

10% Reduction in Turbulent Drag

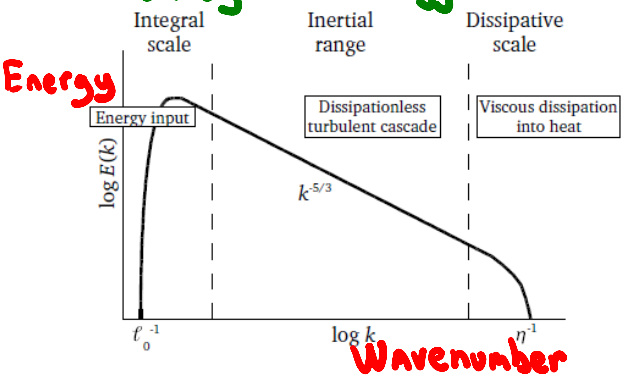
Saves

R$ 20 Billion (US$ 10 Billion)

Statistical Viewpoint

Irregular ⇒ Random, Stochastic

Kolmogorov Energy Cascade



Deterministic Viewpoint

Irregular ⇒ Properties of Solutions

Navier-Stokes Equations

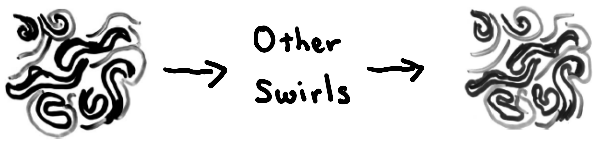
$$\rho\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}\right) = -\nabla p + \nu\nabla^2\vec{v}$$

$$\vec{\nabla}\cdot\vec{v} = 0$$

$\vec{v}$ - velocity

p - pressure

Coherent Structures



Familiar Patterns Recur

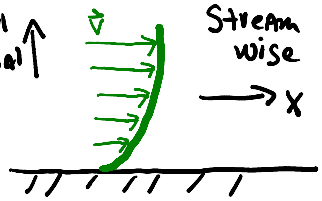
Experimentally Observed for Decades

Recent Theory: Special Navier-Stokes Solutions

Wall-Bounded Shear Flow Turbulence

Pipe Flow

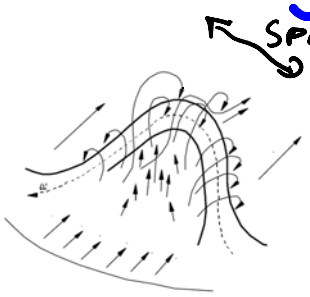
Osborne Reynolds



Boundary Layers ⇉

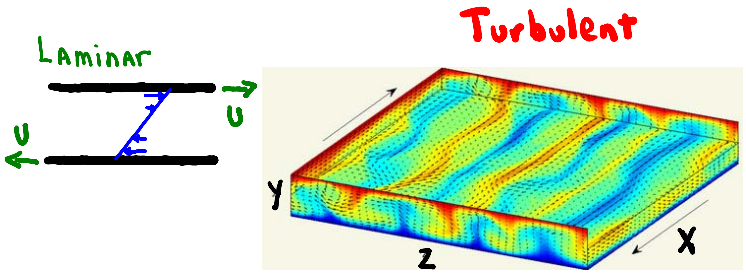
Channel Flows

Coherent Structures in Shear Flow



Hairpin Vortex

Plane Couette Flow



Direct Transition

"Exact Coherent Structures"

Plane Couette

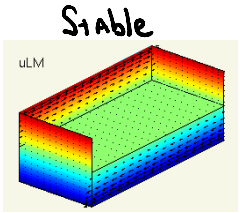
Waleffe (1995)
Nagata (1991)
Kawahara (2001)
Halcrow, Gibson, Cvitanović (2008, 2009)

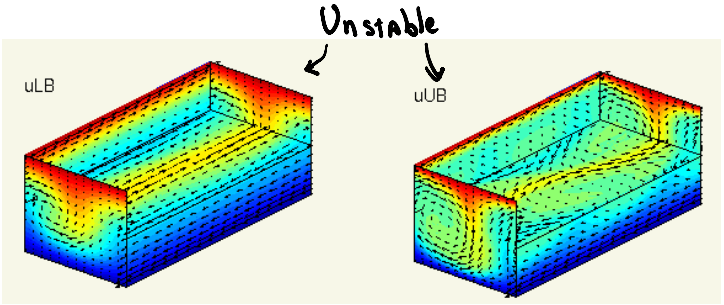
Pipe Flow

Kerswell, Tutty (2007)
Hof et al. (2004)

Exact Solutions

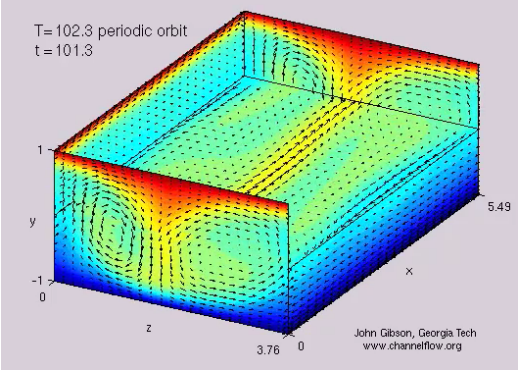
(Time-Independent)



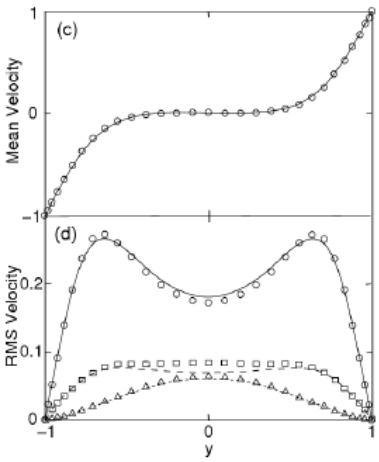


Exact Solutions

(Time-Dependent) Periodic Orbit



Statistical Comparison



o Circles - One Periodic Orbit

— lines Turbulence

State Space Representation

(Phase Space)

$\vec{v}(x,y,z,t)$ gives full specification of state at time t (incompressible, isothermal flow)

Fully Resolved Flow ⇒ Very High Dimensional ($> 10^5$)

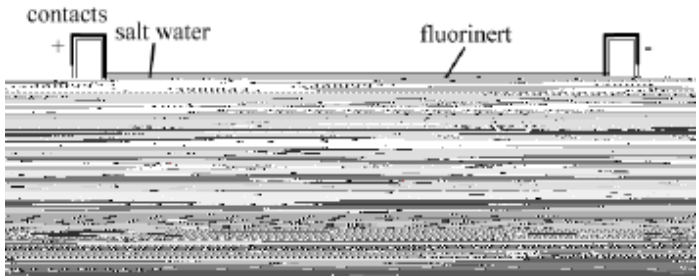
State Space Projection

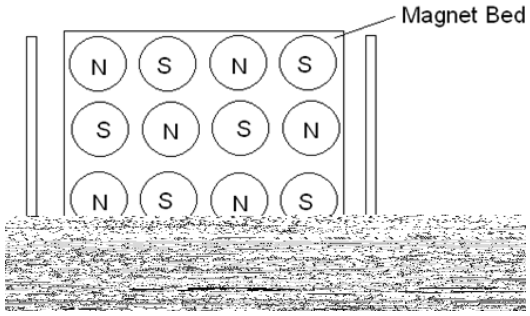
Use exact solutions $\vec{v}_e$ as basis

energy norm $\Rightarrow$ $\vec{a}_e(t) = \frac{1}{V}\int \vec{v}(x,y,z,t)\cdot\vec{v}_e \, dV$

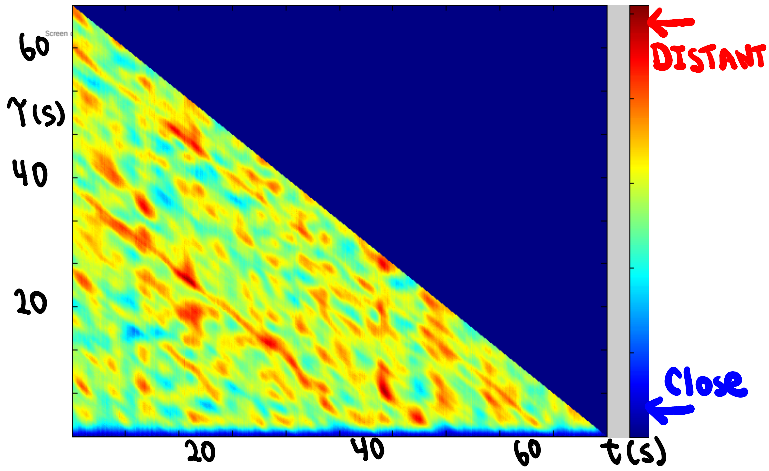
$a_e(t)$ is a coordinate in a reduced space

Visualization: 10^5 → few mensions

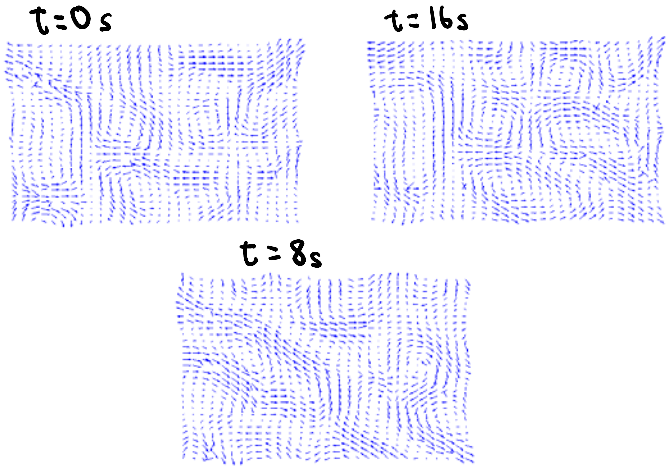

contacts
+
salt water
fluorinert
-


Magnet Bed
N
S
N
S
S
N
S
N
N
S
N
S

Recurrence Plot: 2D Turbulence

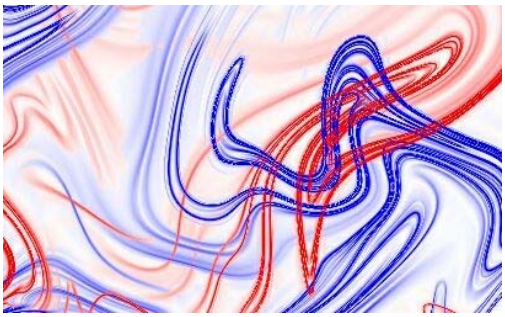


Recurrence in 2D Turbulence?



Lagrangian Coherent Structures
(Stretching Fields)

Governs Turbulent Mixing

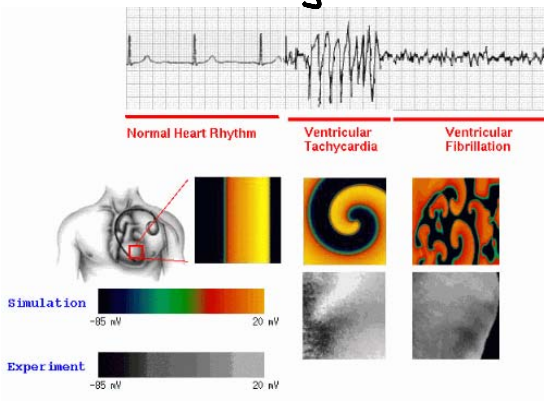


Voth, Haller & Gollub (2002)

Other Applications

Cardiac Arrhythmias

Fenton & Cherry



www.thevirtualheart.org

Summary

- Recent Turbulence Theory

Coherent Structures ⇒ Unstable Exact Solutions

- Experimental Tests Needed

2D Electromagnetic Flows May Provide Good Tests